

Impossibility of obtaining coefficients of thermal expansion of a solid from ultrasonic wave-velocity measurements*

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It is shown that thermal-expansion coefficients of a material cannot be measured by using an ultrasonic cw spectrometer.

In a recent paper Mäntysalo¹ suggested a method of determining thermal-expansion coefficients by using an ultrasonic cw spectrometer. In the present work we show that the suggested method has no validity.

The rationale of the method suggested by Mäntysalo follows: when modulated audio signals of a fixed carrier frequency ν passing through a specimen are fed into a phase-sensitive detector, the recording instrument traces the amplitude variations of uhf signals. As the temperature of the specimen is changed, standing waves are set up due to thermal expansion when the specimen expands through a resonance. Thus, one obtains an oscillating curve of the ultrasonic amplitude as a function of temperature which may be used to calculate the magnitude of the thermal-expansion coefficient $\beta(T)$ of the material. We show that the standing waves set up in the specimen are not due to thermal expansion of the material alone.

Since the frequency ν is kept constant in the experiment suggested in Ref. 1 when resonance occurs at a temperature T

$$L(T) = (n/2)\lambda(T) = (n/2\nu)v(T), \quad (1)$$

where $L(T)$ is the length of the specimen at temperature T , $\lambda(T)$ is the wavelength of the sound wave at temperature T , $v(T)$ is the velocity of a sound wave in the specimen at temperature T , and n is a positive integer. Let the next resonance occur at a temperature T_1 (say, $T_1 > T$), and let there be m wavelengths present in the specimen. Since in general the effect of increasing temperature is to increase the length of a specimen and decrease the magnitude of the elastic wave velocity in the specimen, at a fixed frequency ν we must have $L(T_1) > L(T)$ and $\lambda(T_1) < \lambda(T)$. It, therefore, follows that $m - n > 0$.

In the present case $m - n = +1$, i.e., $\Delta n = 1$. Differentiating Eq. (1) with respect to T and dividing it by $L(T)$ we obtain

$$\begin{aligned} \frac{1}{L(T)} \frac{dL(T)}{dT} &= \beta(T) \\ &= \frac{\nu(T)}{2\nu L(T)} \frac{dn}{dT} + \frac{n}{2\nu} \frac{1}{L(T)} \frac{dv(T)}{dT}, \end{aligned} \quad (2)$$

which may be rewritten as

$$\beta(T) - \gamma(T) = \frac{\nu(T)}{2\nu L(T)} \frac{1}{\Delta T}, \quad (3)$$

where $\gamma(T) = d \ln v(T) / dT$.

Equation (3) suggests that Mäntysalo's method estimates $\beta(T) - \lambda(T)$ and not $\beta(T)$. Thus it is not possible to estimate thermal-expansion coefficients by using an ultrasonic cw spectrometer.

A simpler argument that could be given to show the invalidity of the Mäntysalo's method would be to state that since the ultrasonic measurements give compressibility of a material directly it cannot yield an estimate of thermal-expansion coefficients except under special conditions as shown in Ref. 2.

Finally it is surprising that Mäntysalo obtained the correct values of $\beta(T)$ for Li from the right-hand side expression of Eq. (3), because the ratio $\gamma(T)/\beta(T)$ for Li is of the order of -10 . For example, from the works of Owen and Williams³ and Nash and Smith,⁴ the measured values of $\beta(T)$ and $\gamma(T)$ at 160°K are 4.0×10^{-5} and -39.7×10^{-5} , respectively; whereas the right-hand side of Eq. (3) would yield a correct value of $\beta(T)$ only when $\beta(T) \gg \gamma(T)$. We do not have any reasonable explanation for the above-mentioned contradiction.

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¹E. Mäntysalo, Phys. Lett. **16**, 17 (1965).

²D. P. Dandekar, Phys. Status Solidi A **2**, 769 (1970).

³E. A. Owen and G. I. Williams, Proc. Phys. Soc. Lond. A **67**, 895 (1954).

⁴H. C. Nash and C. S. Smith, J. Phys. Chem. Solids **9**, 113 (1959).